Algebra and Polynomial Theory

Vieta's Formulas

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Definition: Vieta's Formulas

Vieta's formulas give a way of relating the roots of a polynomial with its coefficients. The formulas can be used to solve problems involving the roots of a polynomial, without determining what the roots actually are.

1. Task 1: *Finding Root Sums and Products from a Quadratic* (2 + 2 + 2 Points)

Let a and b be the two roots of the quadratic $2x^2 - 4x - 9$.

a) Use the quadratic formula to find $a + b$
b) Use the quadratic formula to find ab
c) Use the quadratic formula to find $a^2 + b^2$

2. Task 2: *Proof of Vieta's Formula for Quadratics* (5 Points)

Let a, b, and c be real numbers such that $a \neq 0$, and let the quadratic expression $ax^2 + bx + c$ have roots r and s.

Prove that $r + s = -\frac{b}{a}$ and $rs = \frac{c}{a}$.

3. Task 3: *Applying Vieta's Formula to a Quadratic* (2 + 2 + 2 + 2 Points)

Let a and b be the two roots of the quadratic $2x^2 - 4x - 9$. Use Vieta's formula to find:

a) $a + b$
b) ab
c) $a^2 + b^2$
d) $(a + 1)(b + 1)$

4. Task 4: *Computing Root Expressions from Quadratics* (2 + 2 + 2 + 2 Points)

Let r and s be the two roots of the quadratic $x^2 - 7x + 5$. Find:

a) $(r - s)^2$
b) $(r^2 - 1)(s^2 - 1)$

c) $\frac{1}{r} + \frac{1}{s}$
d) $\frac{1}{r^2} + \frac{1}{s^2}$

5. Task 5: *Power Sums and Reciprocal Sums for Quadratics* (2 + 2 + 2 + 2 + 2 + 2 Points)

Let r and s be the two roots of the quadratic $x^2 + 2x - 4$. Find:

a) $r^2 + s^2$
b) $r^3 + s^3$
c) $r^4 + s^4$
d) $\frac{1}{r} + \frac{1}{s}$
e) $\frac{1}{r^2} + \frac{1}{s^2}$
f) $\frac{1}{r^3} + \frac{1}{s^3}$

6. Task 6: *Proof of Vieta's Formula for Cubics* (5 Points)

Let a, b, c, and d be real numbers for which $a \neq 0$, and suppose the cubic $ax^3 + bx^2 + cx + d$ has roots r, s, and t.

Prove that:

$$r + s + t = -\frac{b}{a}$$

$$rs + rt + st = \frac{c}{a}$$

$$rst = -\frac{d}{a}$$

7. Task 7: *Root Relations for Cubics* (2 + 2 + 2 + 2 + 2 + 2 Points)

Let a, b, and c be the three roots of the cubic $8x^3 - 10x^2 - 25x + 15$. Find:

a) $a + b + c$
b) $ab + ac + bc$
c) abc
d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$
e) $a(1 + b + c) + b(1 + c + a) + c(1 + a + b)$
f) $(a + 1)(b + 1)(c + 1)$

8. Task 8: *Symmetric Functions for Cubic Polynomials* (2 + 2 + 2 + 2 + 2 + 2 Points)

Let r, s, and t be the three roots of the cubic $x^3 - 2x^2 - 3x + 4$. Find:

a) $r^2 + s^2 + t^2$
b) $r^3 + s^3 + t^3$
c) $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$
d) $\frac{r+s}{t} + \frac{r+t}{s} + \frac{s+t}{r}$

e) $(r+s)(r+t)(s+t)$
f) $(r^2-1)(s^2-1)(t^2-1)$

9. Task 9: *Advanced Symmetric Functions for Cubics* (2 + 2 + 2 + 2 + 2 Points)

Let r, s, and t be the three roots of the cubic $x^3 - 2x - 1$. Find:

a) $r^2 + s^2 + t^2$
b) $r^3 + s^3 + t^3$
c) $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$
d) $(r+s-t)(r+t-s)(s+t-r)$
e) $(r^2-4)(s^2-4)(t^2-4)$