A Short Guide for A-level Physics: Projectile Motion

Comprehensive study guide for kinematics, angles, and problem-solving

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Contents

Learning Objectives

- Understand the fundamental principles of projectile motion and its component analysis
- Apply kinematic equations to solve motion problems with different launch and landing heights
- Determine optimal launch angles for maximizing range and analyze symmetric properties
- Handle special cases including non-level terrain and interpret real-world effects

1. Kinematics and Basic Assumptions

1.1. Introduction

Projectile motion at A-level treats objects moving near Earth's surface under gravity alone. The analysis involves resolving the initial velocity into horizontal and vertical components, assuming constant gravitational acceleration g, and neglecting air resistance unless explicitly stated. This approach allows the application of constant-acceleration equations independently in each direction.

1.2. Core Concepts and Detailed Explanations

1.2.1. 1. Resolve the Initial Velocity

For an initial speed v_0 launched at angle θ above the horizontal, basic trigonometry of the velocity vector yields:

Definition: Velocity Components

The initial velocity v_0 at angle θ decomposes into:

- Horizontal component: $v_{0x} = v_0 \cos\theta$
- Vertical component: $v_{0y} = v_0 \sin\theta$

These components act independently, allowing separate analysis of horizontal and vertical motion.

1.2.2. 2. Basic Assumptions

The following assumptions underpin the standard projectile motion model:

- Gravity provides constant vertical acceleration of magnitude g (take $g \approx 9.81\ \mathrm{m/s^2}$ unless otherwise specified).
- A consistent sign convention applies throughout (common choice: upward positive, so vertical acceleration $a_y = -g$).
- Air resistance is neglected unless explicitly included in the problem statement.

1.2.3. 3. Component-wise Constant-Acceleration Equations

The kinematic equations apply separately to horizontal (x) and vertical (y) motion. Using u for initial velocity component, v for final component, a for acceleration, s for displacement, and t for time:

General kinematic equations:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Horizontal motion (no horizontal acceleration, $a_x = 0$):

$$v_x = v_{0x} = v_0 \cos\theta$$

$$x = v_{0x}t = v_0 \cos\theta t$$

Vertical motion (vertical acceleration $a_y = -g$ with upward positive):

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt$$

$$y = v_{0y}t - \frac{1}{2}gt^2 = v_0 \sin\theta t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2gy$$

Example 1: Time of Flight and Range

A projectile is launched with $v_0 = 20\,\text{m/s}$ at $\theta = 30^\circ$ above horizontal. Take $g = 9.81\,\text{m/s}^2$.

Resolve:

$$v_{0x} = 20\cos 30^\circ = 20 \times 0.866 = 17.32\,\text{m/s}$$

$$v_{0y} = 20\sin 30^\circ = 20 \times 0.5 = 10.00\,\text{m/s}$$

Time of flight (same launch and landing height):

$$T = \frac{2v_{0y}}{g} = \frac{2 \times 10.00}{9.81} = 2.04\,\text{s}$$

Range:

$$R = v_{0x}T = 17.32 \times 2.04 = 35.3\,\text{m}$$

Result: The projectile lands 35.3 m horizontally after 2.04 seconds of flight.

Important observations:

- Time t is identical for both components and is often determined from vertical motion (e.g., time of flight), then substituted into horizontal displacement to find range.
- For projectiles launched and landing at the same vertical level, time of flight T and range R follow specific formulas (detailed in the next major section).

1.3. Summary of Key Takeaways

Key Concepts

- Resolve initial velocity: $v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$
- Assume constant vertical acceleration $a_y = -g$ and $a_x = 0$; neglect air resistance unless stated
- Apply constant-acceleration equations component-wise:

$$v = u + at$$

,

$$s = ut + \frac{1}{2}at^2$$

,

$$v^2 = u^2 + 2as$$

- Horizontal displacement:

$$x = v_0 \cos\theta \, t$$

- Vertical displacement:

$$y = v_0 \sin\theta \, t - \frac{1}{2}gt^2$$

- Use vertical motion to find time, then horizontal motion to find range

2. Time of Flight, Range, and Maximum Height

2.1. Introduction

Projectile motion under constant gravitational acceleration with initial velocity at an angle exhibits well-defined mathematical relationships. This section derives and applies the standard formulae for time of flight T (when launch and landing occur at the same vertical level), maximum height H above the launch point, and horizontal range R, while highlighting the symmetry properties of the motion.

2.2. Core Concepts and Detailed Explanations

2.2.1. 1. Decompose the Initial Velocity

For launch speed v_0 at angle θ above horizontal:

- Horizontal component: $v_{0x} = v_0 \cos \theta$ (constant, since horizontal acceleration is zero)
- Vertical component: $v_{0y} = v_0 \sin \theta$ (affected by gravity)

Taking upward as positive, the vertical acceleration is $a_y = -g$.

2.2.2. 2. Vertical Motion Equations

The fundamental kinematic equations for vertical motion are:

- Velocity: $v_y(t) = v_{0y} - gt$
- Displacement: $y(t) = v_{0y}t - \frac{1}{2}gt^2$

These assume launch from $y = 0$ and landing at $y = 0$.

2.2.3. 3. Time of Flight T

Setting $y(T) = 0$ to find the nonzero solution for flight time:

$$0 = v_{0y}T - \frac{1}{2}gT^2$$

Factoring:

$$T\left(v_{0y} - \frac{1}{2}gT\right) = 0$$

The nonzero solution is:

$$v_{0y} - \frac{1}{2}gT = 0 \quad \Rightarrow \quad T = \frac{2v_{0y}}{g}$$

Substituting $v_{0y} = v_0 \sin \theta$:

Definition: Time of Flight Formula

$$T = \frac{2v_0 \sin\theta}{g}$$

This formula applies when launch and landing heights are equal. The interpretation is fundamental: the time to rise equals the time to fall, making the total flight time twice the time needed to reach maximum height.

2.2.4. 4. Maximum Height H

At maximum height, the vertical velocity becomes zero: $v_y = 0$. Using the equation $v_y^2 = v_{0y}^2 - 2g\Delta y$ and setting $\Delta y = H$:

$$0 = v_{0y}^2 - 2gH \quad \Rightarrow \quad H = \frac{v_{0y}^2}{2g}$$

Substituting $v_{0y} = v_0 \sin\theta$:

Definition: Maximum Height Formula

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

An alternate derivation uses $t_{\text{up}} = \frac{v_{0y}}{g}$ and the displacement formula to obtain the same result, confirming the height achieved depends on the square of the vertical component.

2.2.5. 5. Horizontal Range R

Range is horizontal speed multiplied by total flight time:

$$R = v_{0x}T$$

Substituting $v_{0x} = v_0 \cos\theta$ and $T = \frac{2v_0 \sin\theta}{g}$:

$$R = v_0 \cos\theta \cdot \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Applying the double-angle identity $2\sin\theta\cos\theta = \sin 2\theta$:

Definition: Range Formula

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

For fixed v_0, the range reaches its maximum when $\sin 2\theta = 1$, which occurs at $\theta = 45^\circ$.

2.2.6. 6. Symmetry of Ascent and Descent

Neglecting air resistance ensures motion is symmetric about the peak:

- The time to rise to a given height equals the time to fall from that height.
- Speeds at a given height during ascent and descent have equal magnitude; vertical velocities are equal in magnitude but opposite in sign.
- Horizontal velocity remains constant throughout the flight.

This symmetry follows from energy conservation or kinematics: v_y^2 depends only on vertical displacement, not on direction of motion.

Example 2: Finding T, H, and R

A projectile is launched with $v_0 = 20\,\mathrm{m/s}$ at $\theta = 30^\circ$. Take $g = 9.81\,\mathrm{m/s^2}$.

Compute:

$$T = \frac{2(20)\sin 30^\circ}{9.81} = \frac{40 \times 0.5}{9.81} \approx 2.04\,\mathrm{s}$$

$$H = \texttt{frac}\{20^2 \sin^2 30^\circ\}\{2(9.81)\} = \texttt{frac}\{400 \times 0.25\}\{19.62\} \approx 5.10\,\mathrm{m}$$

$$R = \texttt{frac}\{20^2 \sin 60^\circ\}\{9.81\} = \texttt{frac}\{400 \times 0.8660\}\{9.81\} \approx 35.3\,\mathrm{m}$$

Result: The projectile reaches 5.10 m, remains airborne for 2.04 s, and travels 35.3 m horizontally.

Exercise 1: Applying the Formulae

Problem: A ball is kicked with speed $v_0 = 12\,\mathrm{m/s}$ at 40°. Calculate the time of flight, maximum height, and range using $g = 9.81\ \mathrm{m/s^2}$.

Hint: Apply the three key formulas: $T = \frac{2v_0 \sin\theta}{g}$, $H = \frac{v_0^2 \sin^2\theta}{2g}$, $R = \frac{v_0^2 \sin 2\theta}{g}$.

Solution:

$$T = \frac{2(12)\sin 40^\circ}{9.81} = \texttt{frac}\{24 \times 0.6428\}\{9.81\} \approx 1.57\,\mathrm{s}$$

$$H = \texttt{frac}\{12^2 \sin^2 40^\circ\}\{2(9.81)\} = \texttt{frac}\{144 \times 0.4132\}\{19.62\} \approx 3.03\,\mathrm{m}$$

$$R = \texttt{frac}\{12^2 \sin 80^\circ\}\{9.81\} = \texttt{frac}\{144 \times 0.9848\}\{9.81\} \approx 14.4\,\mathrm{m}$$

2.3. Summary of Key Takeaways

Key Concepts

- Decompose motion into horizontal ($v_{0x} = v_0 \cos\theta$) and vertical ($v_{0y} = v_0 \sin\theta$) components
- Time of flight (same launch and landing height):
$$T = \frac{2v_0 \sin\theta}{g}$$
- Maximum height:
$$H = \frac{v_0^2 \sin^2\theta}{2g}$$
- Horizontal range (same launch and landing height):
$$R = \frac{v_0^2 \sin 2\theta}{g}$$
- Motion is symmetric about the peak: speeds at equal heights on ascent and descent are equal in magnitude; horizontal velocity is constant

3. Angles, Optimization, and Special Cases

3.1. Introduction

Projectile motion problems frequently ask for the launch angle that maximizes range or investigate how different launch and landing heights affect the optimum angle. This section explains why 45° is optimal for level ground, why complementary angles produce identical ranges, and how to handle special launch and landing geometries using both algebraic and calculus methods.

3.2. Core Concepts and Detailed Explanations

3.2.1. 1. Range on Level Ground

For a projectile launched at speed v_0 and angle θ above horizontal with equal launch and landing heights, the range is:

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

To maximize R with fixed v_0 and g, one maximizes $\sin 2\theta$. Since $\sin x \leq 1$, the maximum occurs when $\sin 2\theta = 1$, which gives $2\theta = 90^\circ$, so:

$$\theta_{\text{opt}} = 45^\circ$$

Alternate justification (calculus): Differentiating $R(\theta) = \frac{v_0^2 \sin 2\theta}{g}$ with respect to θ:

$$\frac{dR}{d\theta} = \frac{v_0^2 \cdot 2\cos 2\theta}{g}$$

Setting this equal to zero gives $\cos 2\theta = 0$, hence $2\theta = 90^\circ$ and $\theta = 45^\circ$. The second derivative confirms this is a maximum.

3.2.2. 2. Complementary Angles

For level ground:

$$R(\theta) = \frac{v_0^2 \sin 2\theta}{g}$$

If one takes the complementary angle $90^\circ - \theta$:

$$R(90^\circ - \theta) = \frac{v_0^2 \sin[2(90^\circ - \theta)]}{g} = \frac{v_0^2 \sin(180^\circ - 2\theta)}{g} = \frac{v_0^2 \sin 2\theta}{g} = R(\theta)$$

Thus, angles θ and $90^\circ - \theta$ yield identical ranges (for instance, 30° and 60°).

⚠ Warning: Complementary Angle Symmetry

The property that $R(\theta) = R(90^{\circ} - \theta)$ holds **only** when launch and landing heights are equal. If heights differ, the trajectory becomes asymmetric and complementary angles no longer produce equal ranges.

Correct application: Always verify that launch and landing are at the same vertical level before invoking this symmetry.

Physical interpretation: A low-angle shot travels farther horizontally for a longer duration but with smaller vertical height. The complementary high-angle shot reaches greater height but carries smaller horizontal speed. These effects trade off exactly when launch and landing heights match, yielding identical ranges.

3.2.3. 3. Non-level Launch and Landing Heights

When launch height $y_0 \neg = 0$ (positive if launch is above landing), the simple range formula no longer applies. Instead, use kinematics directly:

Horizontal position:

$$x(t) = v_0 \cos\theta t$$

Vertical position:

$$y(t) = y_0 + v_0 \sin\theta t - \frac{1}{2} g t^2$$

Find time of flight t_f by solving $y\left(t_f\right) = y_{\text{land}}$ (often ground $= 0$). This yields a quadratic in t_f:

$$-\frac{1}{2} g t_f^2 + v_0 \sin\theta t_f + y_0 - y_{\text{land}} = 0$$

Solve for the positive root $t_f(\theta)$, then obtain range:

$$R(\theta) = v_0 \cos\theta t_f(\theta)$$

To optimize $R(\theta)$, differentiate with respect to θ and solve $\frac{dR}{d\theta} = 0$. The resulting optimal angle generally differs from 45°.

Important limiting behavior:

- If landing height is lower than launch ($y_{\text{land}} < y_0$), the optimal angle is greater than 45°.
- If landing height is higher than launch, the optimal angle is less than 45°.

3.2.4. 4. Limiting and Special Cases

- **Very low angles** ($\theta \to 0^{\circ}$): small vertical component leads to long time near ground, but the range tends to 0 because $\sin 2\theta \to 0$.

- **Very high angles** ($\theta \to 90^\circ$): horizontal speed becomes small, again causing range to approach 0.
- When asked for complementary-angle pairs, always confirm that launch and landing heights are equal; otherwise symmetry fails.
- For targets at the same horizontal level but different heights or with obstacles present, solve vertical motion explicitly and apply constraints.

Example 3: Complementary Angles

With $v_0 = 30\,\text{m/s}$, compare ranges at 30° and 60°.

Since they are complementary, $R(30^\circ) = R(60^\circ)$. Computing the numeric value:

$$R = \frac{30^2 \sin 60^\circ}{g} = \frac{900 \cdot \frac{\sqrt{3}}{2}}{9.81} \approx 79.6\,\text{m}$$

Result: Both launch angles produce the same horizontal range of approximately 79.6 m.

Example 4: Launch Above Landing

Launch from height $y_0 = 10\,\text{m}$ with $v_0 = 20\,\text{m/s}$ at angle θ. Find range as a function of θ.

Solve $0 = 10 + 20\sin\theta t - \frac{1}{2}(9.81)t^2$ for $t > 0$:

$$t = \texttt{frac}\left\{20\sin\theta + \texttt{sqrt}\left\{(20\sin\theta)^2 + 4 \cdot \frac{1}{2}(9.81)\cdot 10\right\}\right\}\{9.81\}$$

(Simplify carefully.) Then:

$$R(\theta) = 20\cos\theta t(\theta)$$

Differentiate numerically or analytically to find the optimal θ (you will find $\theta \approx 50^\circ$, higher than the 45° for level ground).

Result: The optimal angle shifts upward when launching from an elevated position.

3.3. Summary of Key Takeaways

Key Concepts

- For equal launch and landing heights, $R = \frac{v_0^2 \sin 2\theta}{g}$; the optimal angle is $\theta = 45^\circ$ because $\sin 2\theta$ is maximal at unity
- Complementary angles θ and $90^\circ - \theta$ produce equal ranges on level ground
- When launch and landing heights differ, solve the vertical motion quadratic for time of flight, express $R(\theta) = v_0 \cos\theta t_f(\theta)$, and optimize—the optimum generally shifts from 45°
- Use calculus (differentiate $R(\theta)$) or numerical methods to find the optimal angle in non-level scenarios

4. Non-ideal Effects and Problem-solving Strategies

4.1. Introduction

Projectile motion in the idealized case (zero air resistance, constant gravity) constitutes a cornerstone of A-level physics. Real examination problems often include non-ideal features: different launch and landing heights, solving quadratic equations for time, and qualitative reasoning about air resistance or spin effects. This section presents practical methods, worked examples, a problem-solving checklist, common pitfalls, and practice problems.

4.2. Core Concepts and Detailed Explanations

4.2.1. 1. Kinematic Component Equations

Choose axes with x horizontal and y vertical (positive upward). Resolve the initial speed v_0 into components:

$$v_{0x} = v_0 \cos\theta, \qquad v_{0y} = v_0 \sin\theta$$

Assuming constant g (directed downward), the component equations become:

$$x(t) = x_0 + v_{0x} t$$

$$y(t) = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

These form the foundation for all projectile motion analysis.

4.2.2. 2. When Launch and Landing Heights Differ

To find the time(s) of flight, use the vertical equation with the known final height y_f:

$$y_f - y_0 = v_{0y} t - \frac{1}{2} g t^2$$

Rearranging yields a quadratic in t:

$$\frac{1}{2} g t^2 - v_{0y} t + (y_f - y_0) = 0$$

Applying the quadratic formula:

$$t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2g(y_0 - y_f)}}{g}$$

Selecting the physically relevant root:

- If the projectile is launched from y_0 and later lands at y_f, pick the positive root corresponding to the time after launch.
- If both roots are positive (common when landing below launch), choose the larger root for the final landing time.

⚠ Warning: Quadratic Sign Convention

When solving the vertical equation, consistency in sign convention is critical. A common approach is to write:

$$\frac{1}{2}gt^2 - v_{0y}t + (y_f - y_0) = 0$$

and apply the quadratic formula carefully. Errors in setting up or rearranging this quadratic account for many student mistakes.

Correct approach: Always write the vertical equation in standard form before applying the quadratic formula.

4.2.3. 3. Air Resistance and Spin

Unless a specific drag model is provided, keep answers qualitative. Air resistance typically produces the following effects:

Typical effects of air resistance (drag):

- Reduces horizontal speed → shorter range
- Reduces vertical speed faster → lower maximum height
- Breaks time symmetry: ascent is slowed differently from descent → trajectory is not symmetric
- For high speeds, drag scales with speed or speed^2; exact effects depend on model and parameters

Spin (Magnus effect):

- Topspin can push trajectory downward; backspin can produce extra lift (longer time aloft)
- Effects are qualitative unless a lift force model is explicitly given

4.3. Problem-solving Checklist

Following a systematic approach reduces errors:

1. Draw a clear diagram and axes; mark y_0, y_f, v_0, and θ
2. Resolve v_0 into v_{0x} and v_{0y}
3. Write component equations for $x(t)$ and $y(t)$
4. Solve the vertical equation for t (use quadratic formula if needed)
5. Use the correct root for t and compute horizontal displacement $R = v_{0x}t$
6. Check units, signs, and whether the root makes physical sense

4.4. Worked Examples

4.4.1. Example 5: Projectile Launched from a Cliff

A ball is launched with speed $20\,\mathrm{m/s}$ at $\theta = 30^\circ$ from the top of a cliff $y_0 = 15\,\mathrm{m}$ above ground. Find the range along the horizontal to the landing point ($y_f = 0$). Take $g = 9.81\,\mathrm{m/s^2}$.

Resolve components:

$$v_{0x} = 20\cos 30^\circ = 17.32\,\mathrm{m/s}, \qquad v_{0y} = 20\sin 30^\circ = 10.00\,\mathrm{m/s}$$

Solve vertical quadratic for $y_f - y_0 = -15$:

Using the standard form:

$$\frac{1}{2}(9.81)t^2 - 10.00t - 15 = 0$$

Applying the quadratic formula:

$$t = \frac{10.00 \pm \sqrt{394.3}}{2(4.905)} = \frac{10.00 \pm 19.85}{9.81}$$

Two roots arise: $t_1 \approx \frac{10-19.85}{9.81}$ (negative, discard), $t_2 \approx \frac{10+19.85}{9.81} = 2.99\,\mathrm{s}$.

Range:

$$R = v_{0x}t = 17.32 \times 2.99 \approx 51.8\,\mathrm{m}$$

Result: The projectile lands approximately 51.8 m from the cliff base.

Note: Careful setup and sign convention are essential for correct results when solving the quadratic.

4.4.2. Example 6: Symmetric Case for Contrast

If the ball were launched and landed at the same height ($y_f = y_0$), the simpler formula applies directly: $T = \frac{2v_{0y}}{g}$ and $R = v_{0x}T$. This case demonstrates why understanding the symmetric case is valuable for checking algebraic work.

4.5. Common Mistakes and How to Avoid Them

- **Inconsistent sign convention:** Always state the positive direction (typically upward) at the outset.
- **Forgetting to resolve v_0:** Separating velocity into components is essential before applying kinematic equations.
- **Using $T = \frac{2v_{0y}}{g}$ when $y_f \neg = y_0$:** This formula applies only to symmetric launch and landing.
- **Selecting the wrong root:** From the quadratic, discard negative or earlier-crossing roots unless the problem specifically asks for them.
- **Algebraic errors in forming the quadratic:** Prefer writing the standard form

$$\frac{1}{2}gt^2 - v_{0y}t + (y_f - y_0) = 0$$

before using the formula.

Exercise 2: Solving for Non-level Heights

Problem: A projectile is launched at $v_0 = 25\,\text{m/s}$ at $\theta = 40^\circ$ from ground level toward a platform $6\,\text{m}$ above the launch point. Find the time to reach the platform and the horizontal distance traveled.

Hint: Set $y_f - y_0 = 6$, form the quadratic, and choose the positive root.

Solution:

Resolve: $v_{0x} = 25\cos 40^\circ \approx 19.15\,\text{m/s}$ and $v_{0y} = 25\sin 40^\circ \approx 16.07\,\text{m/s}$

Form the quadratic:

$$\frac{1}{2}(9.81)t^2 - 16.07t + 6 = 0$$

Apply the quadratic formula and select the smaller positive root (first passage to height 6 m): $t \approx 0.46\,\text{s}$

Horizontal distance: $x = 19.15 \times 0.46 \approx 8.8\,\text{m}$

4.6. Summary of Key Takeaways

Key Concepts

- Always resolve motion into independent horizontal and vertical components
- For different launch and landing heights, form and solve the quadratic

$$\frac{1}{2}gt^2 - v_{0y}t + (y_f - y_0) = 0$$

- Select the physically meaningful root from the quadratic (typically the larger positive value for landing time)
- Verify that roots make physical sense: time must be positive and correspond to the described motion
- Air resistance qualitatively reduces range, lowers maximum height, and breaks symmetry
- Systematic problem-solving with clear diagrams, component resolution, and consistent sign conventions prevents errors

5. Comprehensive Practice Problems

The following problems integrate concepts across all sections and provide opportunities to develop fluency.

Practice Set A: Basic Kinematics

1. A ball is thrown at $15\,\text{m/s}$ at $45°$. Find time of flight, range, and maximum height. (Take $g = 9.81\,\text{m/s}^2$.)
2. A projectile is fired horizontally from a cliff 20 m high with speed $12\,\text{m/s}$. How far from the cliff base does it land? (Find time from vertical motion and then horizontal displacement.)
3. A projectile is launched so that it lands 50 m away on level ground. If $v_0 = 25\,\text{m/s}$, find the two possible launch angles (neglect air resistance).
4. A particle is launched with $v_0 = 18\,\text{m/s}$ at $\theta = 60°$. Find its velocity (magnitude and direction) after $1.5\,\text{s}$.

Practice Set B: Optimization and Special Cases

1. For $v_0 = 25\,\text{m/s}$ on level ground, find: (a) the angle that maximizes range, (b) the maximum range (take $g = 9.81\,\text{m/s}^2$).
2. Show algebraically that $R(\theta) = R(90° - \theta)$ for level ground.
3. A projectile is launched from an elevated cliff $y_0 = 15\,\text{m}$ with $v_0 = 30\,\text{m/s}$. Derive an expression for $R(\theta)$, then find numerically the angle that gives maximum range.
4. A target lies at horizontal distance D and vertical height h relative to launch. Derive the equation for θ such that the projectile reaches the target; discuss when two real solutions exist and when none exist.

Practice Set C: Non-ideal Effects and Problem-solving

1. A ball thrown at $15\,\text{m/s}$ at $20°$ above horizontal lands at the same height. Find the maximum height and range.
2. Qualitatively, describe three ways air resistance changes a projectile's motion compared to the ideal case.
3. A projectile is launched from ground level at $v_0 = 20\,\text{m/s}$ and $\theta = 35°$. A target platform is located 30 m away horizontally and 8 m above ground. At what height above the platform does the projectile pass if it follows an ideal parabolic path?

Answers (brief):

Set A:

1. $v_{0x} = 10.61\,\text{m/s}$, $v_{0y} = 10.61\,\text{m/s}$; $T \approx 2.16\,\text{s}$; $R \approx 22.9\,\text{m}$; $H \approx 5.74\,\text{m}$.
2. $t = \sqrt{\frac{2\times20}{9.81}} \approx 2.02\,\text{s}$; $x = 12 \times 2.02 \approx 24.2\,\text{m}$.
3. Use $R = \frac{v_0^2 \sin 2\theta}{g}$, so $\sin 2\theta = \frac{Rg}{v_0^2} = 0.7848$; $2\theta \approx 51.8°$ or $128.2°$, giving $\theta \approx 25.9°$ or $64.1°$.
4. $v_x = 18\cos 60° = 9.00\,\text{m/s}$; $v_y = 18\sin 60° - 9.81 \times 1.5 \approx 0.87\,\text{m/s}$; $|v| \approx 9.04\,\text{m/s}$; angle $\approx 5.5°$ above horizontal.

Set B: 1a. $45°$. 1b. $R_{\text{max}} = \frac{v_0^2}{g} \approx 63.6\,\text{m}$.

2. Use identity $\sin(180° - 2\theta) = \sin 2\theta$.

3–4. Follow general method: solve quadratic for t, then maximize or apply discriminant conditions.

6. Summary and Final Remarks

This guide has presented a comprehensive treatment of projectile motion suitable for A-level physics. The core framework consists of resolving initial velocity into components, applying constant-acceleration equations independently to horizontal and vertical directions, and recognizing the symmetry inherent in frictionless motion.

The derived formulae for time of flight, maximum height, and range—when launch and landing heights are equal—are central to examination problems. The geometric insight that a 45° launch angle maximizes range on level ground, and the realization that complementary angles yield equal ranges, deepen understanding of the underlying physics.

For non-ideal scenarios with different launch and landing heights, the approach shifts to solving a quadratic equation for time of flight, then computing range. This method generalizes to any combination of initial and final heights.

While air resistance and spin effects are beyond the scope of this guide, recognizing their qualitative impacts (shorter range, lower peak, asymmetric trajectory) prepares one for more advanced study.

Mastering these techniques through careful problem-solving, attention to sign conventions, and systematic use of the checklist ensures success on examination papers.

Final Key Takeaways

- Projectile motion results from independent horizontal and vertical kinematics under constant gravity
- Component resolution and consistent sign conventions are essential for error-free problem-solving
- Standard formulae apply when launch and landing heights are equal; quadratic equations extend to general cases
- Angle optimization and complementary angle symmetry reveal elegant mathematical structure
- Systematic problem-solving, verification of roots, and qualitative reasoning about non-ideal effects complete the A-level treatment
- Regular practice with varied problems builds conceptual understanding and computational fluency